ANALYSIS OF THE $B \to a_1(1260)$ FORM-FACTORS WITH LIGHT-CONE QCD SUM RULES

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Abstract

In this article, we calculate the $B \to a_1(1260)$ form-factors $V_1(q^2)$, $V_2(q^2)$, $V_3(q^2)$ and $A(q^2)$ with the B-meson light-cone QCD sum rules. Those form-factors are basic parameters in studying the exclusive non-leptonic two-body decays $B \to AP$ and semi-leptonic decays $B \to Al\nu_l$, $B \to A\bar{l}l$. Our numerical results are consistent with the values from the (light-cone) QCD sum rules. The main uncertainty comes from the parameter ω_0 (or λ_B), which determines the shapes of the two-particle and three-particle light-cone distribution amplitudes of the B-meson, it is of great importance to refine this parameter.

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1 Introduction

The weak $B \to P, V, A$ form-factors with $P = \pi, K$, $V = \rho, K^*$ and $A = a_1, K_1$ final states are basic input parameters in studying the exclusive semi-leptonic decays $B \to P(V,A)l\nu_l$, $B \to P(V,A)\bar{l}l$ and radiative decays $B \to V(A)\gamma$, they also determine the factorizable amplitudes in the non-leptonic charmless two-body decays $B \to PP(AP, PV, VV)$. Those decays can be used to determine the CKM matrix elements and to test the standard model, however, it is a great challenge to pin down the uncertainties of the form-factors to obtain more precise results. The exclusive semi-leptonic decays $B \to P(V)l\nu_l$, $B \to P(V)\bar{l}l$ and radiative decays $B \to V\gamma$ and hadronic two-body decays $B \to PP(PV, VV)$ have been studied extensively [1, 2, 3, 4, 5, 6, 7], while the decays $B \to AP, VA$ have been calculated with the QCD factorization approach [8, 9, 10], generalized factorization approach [11, 12], etc. It is more easy to deal with the exclusive semi-leptonic precesses than the non-leptonic precesses, and there have been many works on the relevant form-factors $B \to \pi$, $B \to \rho$ in determining the CKM matrix element V_{ub} [13, 14, 15, 16]. The $B \to a_1(1260)$ form-factors have been studied with the covariant light-front approach [17], ISGW2 quark model [18], quark-meson model [19], QCD sum rules [20], light-cone QCD sum rules [9] and perturbative QCD [21] . However, the values from different theoretical approaches differ greatly from each other.

The BaBar Collaboration and Belle Collaboration have measured the charmless hadronic decays $B^0 \to a_1^{\pm}\pi^{\mp}$ [22, 23]. Moreover, the BaBar Collaboration has

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measured the time-dependent CP asymmetries in the decays $B^0 \to a_1^{\pm}\pi^{\mp}$ with $a_1^{\mp} \to \pi^{\mp}\pi^{\pm}\pi^{\mp}$, from the measured CP parameters, we can determine the decay rates of $a_1^+\pi^-$ and $a_1^-\pi^+$ respectively [24]. Recently, the BaBar Collaboration has reported the observation of the decays $B^{\pm} \to a_1^0\pi^{\pm}, a_1^{\pm}\pi^0$, $B^+ \to a_1^+K^0$ and $B^0 \to a_1^-K^+$ [25, 26]. So it is interesting to re-analyze the $B \to a_1$ form-factors with the B-meson light-cone QCD sum rules [27].

In Ref.[27], the authors obtain new sum rules for the $B \to \pi, K, \rho, K^*$ form-factors from the correlation functions expanded near the light-cone in terms of the B-meson distribution amplitudes, and suggest QCD sum rules motivated models for the three-particle B-meson light-cone distribution amplitudes, which satisfy the relations given in Ref.[28]. In Ref.[28], the authors derive exact relations between the two-particle and three-particle B-meson light-cone distribution amplitudes from the QCD equations of motion and heavy-quark symmetry. The two-particle B-meson light-cone distribution amplitudes have been studied with the QCD sum rules and renormalization group equation [29, 30, 31, 32, 33, 34, 35]. Although the QCD sum rules can't be used for a direct calculation of the distribution amplitudes, it can provide constraints which have to be implemented within the QCD motivated models (or parameterizations) [32].

The B-meson light-cone distribution amplitudes play an important role in the exclusive B-decays, the inverse moment of the two-particle light-cone distribution amplitude $\phi_+(\omega)$ enters many factorization formulas (for example, see Refs.[3, 4]). However, the light-cone distribution amplitudes of the B-meson are received relatively little attention comparing with the ones of the light pseudoscalar mesons and vector mesons, our knowledge about the nonperturbative parameters which determine those light-cone distribution amplitudes is limited and an additional application (or estimation) based on QCD is useful.

In this article, we use the B-meson light-cone QCD sum rules to study the $B \to a_1$ form-factors. The semi-leptonic decays $B \to Al\nu_l$ can be observed at the LHCb, where the $b\bar{b}$ pairs will be copiously produced with the cross section about $500\,\mu b$.

We can also study the form-factors with the light-cone QCD sum rules using the light-cone distribution amplitudes of the axial-vector mesons. Recently, the twist-2 and twist-3 light-cone distribution amplitudes of the axial-vector mesons have been calculated with the QCD sum rules [36].

The B-meson light-cone QCD sum rules have given reasonable values for the $B \to \pi, K, \rho, K^*$ form-factors [27], so it is interesting to study the $B \to a_1$ form-factors and cross-check the properties of the B-meson light-cone distribution amplitudes. Furthermore, it is necessary to investigate the form-factors with different approaches and compare the predictions of different approaches.

The article is arranged as: in Section 2, we derive the $B \to a_1(1260)$ form-factors with the light-cone QCD sum rules; in Section 3, the numerical result and discussion; and Section 4 is reserved for conclusion.

2 $B \to a_1(1260)$ form-factors with light-cone QCD sum rules

In the following, we write down the definitions for the weak form-factors $V_1(q^2)$, $V_2(q^2)$, $V_3(q^2)$, $V_0(q^2)$ and $A(q^2)$ [17],

$$
\begin{aligned}
\langle a_1(p)|J_\mu(0)|B(P)\rangle &= i\left\{(M_B-M_a)\epsilon^*_\mu V_1(q^2)-\frac{\epsilon^*\cdot P}{M_B-M_a}(P+p)_\mu V_2(q^2)\right. \\
&\quad \left. -2M_a\frac{\epsilon^*\cdot P}{q^2}q_\mu[V_3(q^2)-V_0(q^2)]\right\}\,, \qquad (1)\\
\langle a_1(p)|J^A_\mu(0)|B(P)\rangle &= \frac{1}{M_B-M_a}\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu(P+p)_\alpha q_\beta A(q^2)\,, \qquad (2)
\end{aligned}
$$

where

$$
\begin{aligned}
V_3(q^2) &= \frac{M_B-M_a}{2M_a}V_1(q^2)-\frac{M_B+M_a}{2M_a}V_2(q^2)\,,\\
J_\mu(x) &= \bar{d}(x)\gamma_\mu b(x)\,,\\
J^A_\mu(x) &= \bar{d}(x)\gamma_\mu\gamma_5 b(x)\,, \qquad (3)
\end{aligned}
$$

$V_0(0) = V_3(0)$, and the ϵ_μ is the polarization vector of the axial-vector meson $a_1(1260)$. We study the weak form-factors $V_1(q^2)$, $V_2(q^2)$, $V_3(q^2)$, $V_0(q^2)$ and $A(q^2)$ with the two-point correlation functions $\Pi^i_\mu(p,q)$,

$$
\begin{aligned}
\Pi^i_{\mu\nu}(p,q) &= i\int d^4x\, e^{ip\cdot x}\langle 0|T\left\{J^a_\mu(x)J^i_\mu(0)\right\}|B(P)\rangle\,,\\
J^a_\mu(x) &= \bar{u}(x)\gamma_\mu\gamma_5 d(x)\,, \qquad (4)
\end{aligned}
$$

where $J^i_\mu(x) = J_\mu(x)$ and $J^A_\mu(x)$ respectively, and the axial-vector current $J^a_\mu(x)$ interpolates the axial-vector meson $a_1(1260)$. The correlation functions $\Pi^i_\mu(p,q)$ can be decomposed as

$$
\begin{aligned}
\Pi^1_\mu(p,q) &= \Pi_A g_{\mu\nu}+\Pi_B q_\mu p_\nu+\Pi_C p_\mu q_\nu+\Pi_D q_\mu q_\nu+\Pi_D p_\mu p_\nu\,,\\
\Pi^2_\mu(p,q) &= \Pi_2\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta+\cdots \qquad (5)
\end{aligned}
$$

due to Lorentz covariance. In this article, we derive the sum rules with the tensor structures $g_{\mu\nu}$, $q_\mu p_\nu$ and $\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta$ respectively to avoid contaminations from the π meson.

According to the basic assumption of current-hadron duality in the QCD sum rules approach [37, 38], we can insert a complete series of intermediate states with the same quantum numbers as the current operator $J^a_\mu(x)$ into the correlation functions $\Pi^i_\mu(p,q)$ to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the meson $a_1(1260)$, the correlation functions

$\Pi^i_{\mu\nu}(p,q)$ can be expressed in the following form,

$$
\begin{aligned}
\Pi^1_{\mu\nu}(p,q) &= -\frac{if_aM_a(M_B-M_a)V_1(q^2)}{M_a^2-p^2}g_{\mu\nu}+ \\
&\quad \frac{2if_aM_aV_2(q^2)}{(M_B-M_a)(M_a^2-p^2)}q_\mu p_\nu+\cdots\,, \qquad (6)\\
\Pi^2_{\mu\nu}(p,q) &= \frac{2f_aM_aA(q^2)}{(M_B-M_a)(M_a^2-p^2)}\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta+\cdots\,, \qquad (7)
\end{aligned}
$$

where we have used the standard definition for the decay constant f_a, $\langle 0|J^a_\mu(0)|a_1(p)\rangle = f_aM_a\epsilon_\mu$.

In the following, we briefly outline the operator product expansion for the correlation functions $\Pi^i_\mu(p,q)$ in perturbative QCD theory. The calculations are performed at the large space-like momentum region $p^2 \ll 0$ and $0 \le q^2 < m_b^2 + m_bp^2/\bar{\Lambda}$, where $M_B = m_b + \bar{\Lambda}$ in the heavy quark limit. We write down the propagator of a massless quark in the external gluon field in the Fock-Schwinger gauge and the light-cone distribution amplitudes of the B meson firstly [39],

$$
\begin{aligned}
\langle 0|T\{q_i(x_1)\,\bar{q}_j(x_2)\}|0\rangle &= i\int\frac{d^4k}{(2\pi)^4}e^{-ik(x_1-x_2)} \\
&\quad \left\{\frac{\not k}{k^2}\delta_{ij}-\int_0^1 dv G^{ij}_{\mu\nu}(vx_1+(1-v)x_2)\right. \\
&\quad \left.\left[\frac{1}{2}\frac{\not k}{k^4}\sigma^{\mu\nu}-\frac{1}{k^2}v(x_1-x_2)^\mu\gamma^\nu\right]\right\}\,, \\
\langle 0|\bar{q}_\alpha(x)h_{v\beta}(0)|B(v)\rangle &= -\frac{if_Bm_B}{4}\int_0^\infty d\omega e^{-i\omega v\cdot x} \\
&\quad \left\{(1+\not v)\left[\phi_+(\omega)-\frac{\phi_+(\omega)-\phi_-(\omega)}{2v\cdot x}\not x\right]\gamma_5\right\}_{\beta\alpha}\,, \\
\langle 0|\bar{q}_\alpha(x)G_{\lambda\rho}(ux)h_{v\beta}(0)|B(v)\rangle &= \frac{f_Bm_B}{4}\int_0^\infty d\omega\int_0^\infty d\xi e^{-i(\omega+u\xi)v\cdot x} \\
&\quad \left\{(1+\not v)\left[(v_\lambda\gamma_\rho-v_\rho\gamma_\lambda)\Big(\Psi_A(\omega,\xi)-\Psi_V(\omega,\xi)\Big)\right.\right. \\
&\quad -i\sigma_{\lambda\rho}\Psi_V(\omega,\xi)-\frac{x_\lambda v_\rho-x_\rho v_\lambda}{v\cdot x}X_A(\omega,\xi) \\
&\quad \left.\left.+\frac{x_\lambda\gamma_\rho-x_\rho\gamma_\lambda}{v\cdot x}Y_A(\omega,\xi)\right]\gamma_5\right\}_{\beta\alpha}\,, \qquad (8)
\end{aligned}
$$

where

$$\begin{aligned}
\phi_+(\omega) &= \frac{\omega}{\omega_0^2}e^{-\frac{\omega}{\omega_0}}\,, \;\; \phi_-(\omega)=\frac{1}{\omega_0}e^{-\frac{\omega}{\omega_0}}\,, \\
\Psi_A(\omega,\xi) &= \Psi_V(\omega,\xi)=\frac{\lambda_E^2}{6\omega_0^4}\xi^2 e^{-\frac{\omega+\xi}{\omega_0}}\,, \\
X_A(\omega,\xi) &= \frac{\lambda_E^2}{6\omega_0^4}\xi(2\omega-\xi)e^{-\frac{\omega+\xi}{\omega_0}}\,, \\
Y_A(\omega,\xi) &= -\frac{\lambda_E^2}{24\omega_0^4}\xi(7\omega_0-13\omega+3\xi)e^{-\frac{\omega+\xi}{\omega_0}}\,,
\end{aligned} \tag{9}$$

the ω_0 and λ_E^2 are some parameters of the B-meson light-cone distribution amplitudes.

Substituting the d quark propagator and the corresponding B-meson light-cone distribution amplitudes into the correlation functions $\Pi^i_\mu(p,q)$, and completing the integrals over the variables x and k, finally we obtain the representation at the level of quark-gluon degrees of freedom. In this article, we take the three-particle B-meson light-cone distribution amplitudes suggested in Ref.[27], they obey the powerful constraints derived in Ref.[28] and the relations between the matrix elements of the local operators and the moments of the light-cone distribution amplitudes, if the conditions $\omega_0=\frac{2}{3}\bar{\Lambda}$ and $\lambda_E^2=\lambda_H^2=\frac{3}{2}\omega_0^2=\frac{2}{3}\bar{\Lambda}^2$ are satisfied [29].

In the region of small ω, the exponential form of distribution amplitude $\phi_+(\omega)$ is numerically close to the more elaborated model (or the BIK distribution amplitude (BIK DA)) suggested in Ref.[32],

$$\phi_+(\omega,\mu=1\mathrm{GeV})=\frac{4\omega}{\pi\lambda_B(1+\omega^2)}\left[\frac{1}{1+\omega^2}-2\frac{\sigma_B-1}{\pi^2}\ln\omega\right]\,, \tag{10}$$

where $\omega_0=\lambda_B$. The parameters λ_B and σ_B are determined from the heavy quark effective theory QCD sum rules including the radiative and nonperturbative corrections. There are other phenomenological models for the two-particle B-meson light-cone distribution amplitudes, for example, the k_T factorization formalism [40, 41], in this article, we use the QCD sum rules motivated models.

After matching with the hadronic representation below the continuum threshold s_0, we obtain the following three sum rules for the weak form-factors $V_1(q^2)$, $V_2(q^2)$

and $A(q^2)$ respectively,

$$
\begin{aligned}
V_1(q^2) &= \frac{1}{f_a M_a (M_B - M_a)} e^{\frac{M_a^2}{M^2}} \left\{ -\frac{1}{2} f_B M_B M^2 \int_0^{\sigma_0} d\sigma \phi_+(\omega') \frac{d}{d\sigma} e^{-\frac{s}{M^2}} \right. \\
& -\frac{f_B M_B}{2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^{\infty} \frac{d\xi}{\xi} [\Psi_A(\omega,\xi) - \Psi_V(\omega,\xi)] \frac{d}{d\sigma} \frac{1}{\bar{\sigma}} e^{-\frac{s}{M^2}} \\
& + \frac{f_B M_B^2}{M^2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^{\infty} \frac{d\xi}{\xi} \frac{(1-2u)[3\widetilde{X}_A(\omega,\xi) - 2\widetilde{Y}_A(\omega,\xi)]}{\bar{\sigma}^2} e^{-\frac{s}{M^2}} \\
& - f_B \int_0^{\sigma_0} d\sigma \int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^{\infty} \frac{d\xi}{\xi} \frac{(1-2u)\widetilde{X}_A(\omega,\xi)}{\bar{\sigma}^3} e^{-\frac{s}{M^2}} \\
& \left. \left[\frac{\widetilde{M}_B^4 - 4 s M_B^2}{2M^4} - 2 \frac{\widetilde{M}_B^2 - 2M_B^2}{M^2} + 1 \right] \right\} , \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
V_2(q^2) &= \frac{M_B - M_a}{2 f_a M_a} e^{\frac{M_a^2}{M^2}} \left\{ f_B M_B \int_0^{\sigma_0} d\sigma \left[\phi_+(\omega') \frac{1-2\sigma}{\bar{\sigma}} + \right. \right. \\
& \left. \frac{2M_B}{M^2} [\widetilde{\phi}_+(\omega') - \widetilde{\phi}_-(\omega')] \frac{\sigma}{\bar{\sigma}} \right] e^{-\frac{s}{M^2}} \\
& + \frac{f_B M_B}{M^2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^{\infty} \frac{d\xi}{\xi} \frac{(2\sigma - 3)[\Psi_A(\omega,\xi) - \Psi_V(\omega,\xi)]}{\bar{\sigma}^2} e^{-\frac{s}{M^2}} \\
& + \frac{f_B}{M^2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^{\infty} \frac{d\xi}{\xi} (1-2u) \widetilde{X}_A(\omega,\xi) (6 + \frac{d}{d\sigma}) \frac{1}{\bar{\sigma}^2} e^{-\frac{s}{M^2}} \\
& - \frac{4 f_B M_B^2}{M^4} \int_0^{\sigma_0} d\sigma \int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^{\infty} \frac{d\xi}{\xi} (1-2u) \widetilde{Y}_A(\omega,\xi) \frac{\sigma}{\bar{\sigma}^2} e^{-\frac{s}{M^2}} \\
& - \frac{4 f_B}{M^2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^{\infty} \frac{d\xi}{\xi} \frac{(1-2u)\widetilde{X}_A(\omega,\xi)}{\bar{\sigma}^3} \\
& \left. \left[2 - \sigma - \frac{2s - \sigma \widetilde{M}_B^2}{2M^2} \right] e^{-\frac{s}{M^2}} \right\} , \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
A(q^2) &= \frac{M_B - M_a}{2 f_a M_a} e^{\frac{M_a^2}{M^2}} \left\{ f_B M_B \int_0^{\sigma_0} d\sigma \frac{\phi_+(\omega')}{\bar{\sigma}} e^{-\frac{s}{M^2}} \right. \\
& + \frac{f_B M_B}{M^2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^{\infty} \frac{d\xi}{\xi} \frac{[\Psi_A(\omega,\xi) - \Psi_V(\omega,\xi)]}{\bar{\sigma}^2} e^{-\frac{s}{M^2}} \\
& \left. + \frac{f_B}{M^2} \int_0^{\sigma_0} d\sigma \int_0^{\sigma M_B} d\omega \int_{\sigma M_B - \omega}^{\infty} \frac{d\xi}{\xi} (1-2u) \widetilde{X}_A(\omega,\xi) \frac{d}{d\sigma} \frac{1}{\bar{\sigma}^2} e^{-\frac{s}{M^2}} \right\} , \qquad (13)
\end{aligned}
$$

where

$$
\begin{aligned}
s &= M_B^2\sigma - \frac{\sigma}{\bar\sigma}q^2\,, \;\; \omega' = \sigma M_B\,, \;\; \bar\sigma = 1-\sigma\,, \\
\sigma_0 &= \frac{s_0 + M_B^2 - q^2 - \sqrt{(s_0+M_B^2-q^2)^2 - 4s_0M_B^2}}{2M_B^2}\,, \\
u &= \frac{\sigma M_B - \omega}{\xi}\,, \;\; \widetilde{M}_B^2 = M_B^2(1+\sigma) - \frac{1}{\bar\sigma}q^2\,, \\
\widetilde{X}_A(\omega,\xi) &= \int_0^\omega d\lambda X_A(\lambda,\xi)\,, \;\; \widetilde{Y}_A(\omega,\xi) = \int_0^\omega d\lambda Y_A(\lambda,\xi)\,, \\
\widetilde{\phi}_\pm(\omega) &= \int_0^\omega d\lambda \phi_\pm(\lambda)\,. \qquad (14)
\end{aligned}
$$

In Ref.[31], Lange and Neubert observe that the evolution effects drive the light-cone distribution amplitude $\phi_+(\omega)$ toward a linear growth at the origin and generate a radiative tail that falls off slower than $\frac{1}{\omega}$, even if the initial function has an arbitrarily rapid falloff, which implies the normalization integral of the $\phi_+(\omega)$ is ultraviolet divergent. In this article, we derive the sum rules without the radiative $\mathcal{O}(\alpha_s)$ corrections, the ultraviolet behavior of the $\phi_+(\omega)$ plays no role at the leading order ($\mathcal{O}(1)$). Furthermore, the duality thresholds in the sum rules are well below the region where the effect of the tail becomes noticeable. The nontrivial renormalization of the B-meson light-cone distribution amplitude is so far known only for the $\phi_+(\omega)$, we use the light-cone distribution amplitudes of order $\mathcal{O}(1)$, which satisfy all QCD constraints.

3 Numerical result and discussion

The input parameters are taken as $\omega_0 = \lambda_B(\mu) = (0.46 \pm 0.11)\,\mathrm{GeV}$, $\mu = 1\,\mathrm{GeV}$ [32], $\lambda_E^2 = (0.11 \pm 0.06)\,\mathrm{GeV}^2$ [29], $M_a = (1.23 \pm 0.06)\,\mathrm{GeV}$, $f_a = (0.238 \pm 0.010)\,\mathrm{GeV}$, $s_0 = (2.55 \pm 0.15)\,\mathrm{GeV}^2$ [36], $M_B = 5.279\,\mathrm{GeV}$, $f_B = (0.18 \pm 0.02)\,\mathrm{GeV}$ [42, 43].

The Borel parameters in the three sum rules are taken as $M^2 = (1.1-1.5)\,\mathrm{GeV}^2$, in this region, the values of the weak form-factors $V_1(q^2)$, $V_2(q^2)$ and $A(q^2)$ are stable enough.

Taking into account all the uncertainties, we obtain the numerical values of the weak form-factors $V_1(q^2)$, $V_2(q^2)$ and $A(q^2)$, which are shown in Fig.1, at zero momentum transfer,

$$
\begin{aligned}
V_1(0) &= 0.67^{+0.33}_{-0.21}\,, \\
V_2(0) &= 0.31^{+0.18}_{-0.11}\,, \\
V_3(0) &= 0.29^{+0.07}_{-0.06}\,, \\
V_0(0) &= 0.29^{+0.07}_{-0.06}\,, \\
A(0) &= 0.41^{+0.20}_{-0.13}\,. \qquad (15)
\end{aligned}
$$

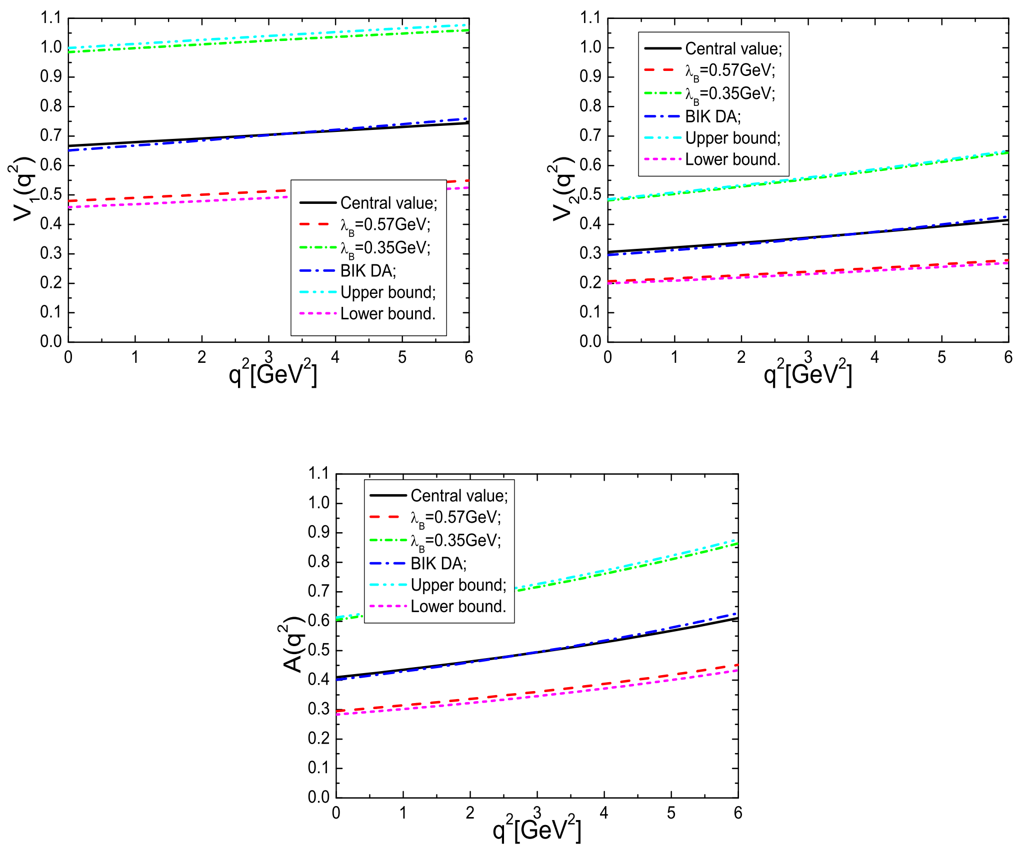

Figure 1: The form-factors $V_1(q^2)$, $V_2(q^2)$ and $A(q^2)$ with the momentum transfer q^2 .

theoretical approaches	$V_0(0)$
Covariant light front approach [17]	0.13
ISGW2 quark model [18]	1.01
quark-meson model [19]	1.20
QCD sum rules [20]	0.23 ± 0.05
perturbative QCD [21]	$0.34^{+0.07+0.08}_{-0.06-0.08}$
light-cone sum rules [9]	0.30 ± 0.05
This work (light-cone sum rules)	$0.29^{+0.07}_{-0.06}$

Table 1: The form-factor $V_0(0)$ from different theoretical approaches. I know the updated value 0.30 ± 0.05 from private communication with Prof. H.Y.Cheng, their work is still in progress.

theoretical approaches	$A(0)$
Covariant light front approach [17]	0.25
quark-meson model [19]	0.09
QCD sum rules [20]	0.42 ± 0.06
perturbative QCD [21]	$0.26^{+0.06+0.03}_{-0.05-0.03}$
This work (light-cone sum rules)	$0.41^{+0.20}_{-0.13}$

Table 2: The form-factor $A(0)$ from different theoretical approaches.

The form-factors can be parameterized in the double-pole form,

$$F_i(q^2) = \frac{F_i(0)}{1 + a_F q^2/M_b^2 + b_F q^4/M_B^4}, \qquad (16)$$

where we use the notation $F_i(q^2)$ to denote the $V_1(q^2)$, $V_2(q^2)$ and $A(q^2)$, the a_F and b_F are the corresponding coefficients and their values are presented in Table 3.

In calculation, we observe the dominating contributions in the three sum rules come from the two-particle B-meson light-cone distribution amplitudes, the contributions from the three-particle B-meson light-cone distribution amplitudes are of minor importance, about 1%, and can be neglected safely. It is not un-expected that the main uncertainty comes from the parameter ω_0 (or λ_B), which determines the shapes of the two-particle and three-particle light-cone distribution amplitudes of the B meson. From Fig.1, we can see that the uncertainty of the parameter λ_B almost saturates the total uncertainties, it is of great importance to refine this parameter. In this article, we take the value from the QCD sum rules in Ref.[32], where the B-meson light-cone distribution amplitude ϕ_+ is parameterized by the matrix element of the bilocal operator at imaginary light-cone separation.

In the region of small ω, the exponential (Gaussian) form of distribution amplitude $\phi_+(\omega)$ is numerically close to the BIK DA suggested in Ref.[32]. In Fig.1, we also present the numerical results with the BIK DA for the central values of the input parameters λ_B and σ_B, the Gaussian distribution amplitude and the BIK DA

	a_F	b_F
$V_1(q^2)$	-0.518	0.159
$V_2(q^2)$	-1.330	0.532
$A(q^2)$	-1.649	0.561

Table 3: The parameters for the fitted form-factors.

lead to almost the same values.

From Table 1, we can see that the values of the $V_0(0)$ from the covariant light-front approach, ISGW2 quark model and quark-meson model differ greatly from the corresponding ones from the (light-cone) QCD sum rules, while the values from the (light-cone) QCD sum rules and perturbative QCD are consistent with each other. From Table 2, we observe that the values of the $A(0)$ from the covariant light-front approach, quark-meson model and perturbative QCD differ greatly from the corresponding ones from the (light-cone) QCD sum rules, while the values of the form-factors from the (light-cone) QCD sum rules are consistent with each other.

4 Conclusion

In this article, we calculate the weak form-factors $V_1(q^2)$, $V_2(q^2)$, $V_3(q^2)$ and $A(q^2)$ with the B-meson light-cone QCD sum rules. The form-factors are basic parameters in studying the exclusive hadronic two-body decays $B \to AP$ and semi-leptonic decays $B \to Al\nu_l$, $B \to Al\bar{l}$. Our numerical values are consistent with the values from the (light-cone) QCD sum rules. The main uncertainty comes from the parameter ω_0 (or λ_B), which determines the shapes of the two-particle and three-particle light-cone distribution amplitudes of the B meson, it is of great importance to refine this parameter. However, it is a difficult work, as we cannot extract the values of the basic parameter λ_B directly from the experimental data on the semi-leptonic decays $B \to Al\nu_l$.

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